

Radix and Reduced Radix Complementation

Nearly all computers use some form of number complementation to perform tasks. Using complementary numbers is faster and simpler than using signed numbers (numbers with a "+" or "-" sign) in computer operations. Therefore, it is important to understand the radix and reduced radix complements of numbers when studying computer science.

Note: Radix and reduced radix complements are sometimes referred to as b's complements and (b-1)'s complements, where b is the base of the numbers being complemented. Therefore, the reader may notice radix complements referred to as 2's, 8's, 10's, or 16's complements and reduced radix complements as 1's, 7's, 9's, or 15's complements.

Radix Complements (b's Complements)

Definition: Two numbers X and Y are considered to be radix complements of each other if:

$$X + Y = 10_b^n$$

Where n is the number of digits in X and Y, and
Where b is the base of X and Y

Consider the case where X is some base b number, six digits long:

$$X = \text{*****}_b$$

Then Y is also a base b number six digits long, but satisfying the condition

$$\begin{array}{r} X = \text{*****}_b \\ + Y = \text{*****}_b \\ \hline 100000_b = 10_b^6 \end{array}$$

Here, X is the radix complement of Y, and Y is the radix complement of X.

Some examples of radix complements are the following:

101011 ₂ and 010101 ₂	because	101011 ₂ + 010101 ₂	=	10 ₂ ¹¹⁰
372 ₈ and 406 ₈	because	372 ₈ + 406 ₈	=	10 ₈ ⁴
9046 ₁₀ and 0954 ₁₀	because	9046 ₁₀ + 0954 ₁₀	=	10 ₁₀ ⁴
A62E3 ₁₆ and 59DIC ₁₆	because	A62E3 ₁₆ + 59DIC ₁₆	=	10 ₁₆ ⁵

Reduced Radix Complements (b-1's Complements)

Reduced radix complements are very similar to radix complements

Definition: Two numbers X and Y are considered to be reduced radix complements of each other if:

$$X + Y = 10_b^n - 1$$

Where n is the number of digits in X and Y, and

Where b is the base of X and Y

For example, 1725_{10} and 8274_{10} are reduced radix complements of each other because:

$$\begin{array}{r} 1725_{10} \\ + 8274_{10} \\ \hline 9999_{10} \end{array}$$

Therefore, the reader should notice that if X and Y are numbers n digits long and are in base b, their sum should equal a number, n digits long, where (b-1) is every digit.

This means the sum should equal n number of 9's in base 10, n number of 1's in base 2, n number of 7's base 8 and n number of F's base 16.

Some examples of reduced radix complements are the following:

$$\begin{array}{llll} 10110_2 \text{ and } 01001_2 & \text{because} & 10110_2 + 01001_2 = & 11111_2 = 10_2^{101} - 1 \\ 063_8 \text{ and } 714_8 & \text{because} & 063_8 + 714_8 = & 777_8 = 10_8^3 - 1 \\ 5874_{10} \text{ and } 4125_{10} & \text{because} & 5874_{10} + 4125_{10} = & 9999_{10} = 10_{10}^4 - 1 \end{array}$$

Bibliography

Early, G. (1995). Mathematics 2358: Mathematics Supplement. San Marcos, TX: SWT Print Shop.

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